

Termination and Complexity
Notations for Rewriting
Intl. School on Rewriting 2014

Johannes Waldmann (HTWK Leipzig)

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Signature, Terms

signature: set of function symbols with arities

example: $\Sigma = \{f/2, S/1, Z/0\}$.

term t over signature Σ : $t = f(t_1, \dots, t_k)$ where $(f/k) \in \Sigma$ and $\forall 1 \leq i \leq k : t_i \in \text{Term}(\Sigma)$

example: $f(S(Z()), Z()) \in \text{Term}(\Sigma)$.

Positions, Subterms

positions in t :

$$\text{Pos}(f(t_1, \dots, t_k)) = \{\square\} \cup \{[i] \circ p \mid 1 \leq i \leq k, p \in \text{Pos}(t_i)\}$$

example:

$$\text{Pos}(f(S(Z()), Z())) = \{\square, [1], [1, 1], [2]\}$$

subterm at position: for $p \in \text{Pos}(t)$:

$$t(\square) = t, f(t_1, \dots, t_k)([i] \circ p) = t_i(p)$$

function symbol at position: for $p \in \text{Pos}(t)$:

$$f(\dots)(\square) = f, f(t_1, \dots, t_k)([i] \circ p) = t_i(p)$$

(note: overloaded notation)

Positions, Replacements at position

applications:

subterm relation: $s \trianglelefteq t \text{ iff } \exists p \in \text{Pos}(t) : s = t(p)$

size of term: $|t| = |\text{Pos}(t)|$

$t(p := s')$ for $p \in \text{Pos}(t)$:

$t([\] := s) = s, f(\dots, t_i, \dots)([i] \circ p := s) =$

$f(\dots, t_i(p := s), \dots)$.

Variables, Substitutions

term over signature Σ with variables from set X
(disjoint from Σ): $\text{Term}(\Sigma, X)$:

$x \in X \Rightarrow x \in \text{Term}(\Sigma, X)$ and

$\forall i : t_i \in \text{Term}(\Sigma, X) \Rightarrow f(t_1, \dots, t_k) \in \text{Term}(\Sigma, X)$

ground substitution: partial mapping

$\sigma : X \hookrightarrow \text{Term}(\Sigma)$

extended to mapping

$\sigma : \text{Term}(\Sigma, X) \rightarrow \text{Term}(\Sigma)$, written post-fix

by $x\sigma = \sigma(x)$, $f(t_1, \dots, t_k)\sigma = f(t_1\sigma, \dots, t_k\sigma)$

Term Rewriting

rule $(l, r) \in \text{Term}(\Sigma, X)^2$

apply rule (l, r) at position p in term t

$t \xrightarrow{(l,r),p} t'$ if

▶ $p \in \text{Pos}(t)$,

▶ $\exists \sigma : t(p) = l\sigma \wedge t(p := r\sigma) = t'$

rewrite system R is set of rules

$t \rightarrow_R t'$ iff $\exists (l, r) \in R, p : t \xrightarrow{(l,r),p} t'$

String Rewriting as a special case

- ▶ all symbols are unary
- ▶ there is one nullary symbol “0” that never occurs in rules

example $R = \{a(b(x)) \rightarrow b(a(x))\}$ over signature $\Sigma = \{a/1, b/1, 0/0\}$

- ▶ terms are in fact strings:
 $a(b(a(b(0)))) \sim abab \in \{a, b\}^*$
- ▶ a rule is a pair of strings,
 $a(b(x)) \rightarrow b(a(x)) \sim ab \rightarrow ba$
- ▶ string rewriting system $R \subseteq (\Sigma^*)^2$
- ▶ string rewriting relation: $w \rightarrow_R w' \iff \exists (l, r) \in R, p, q \in \Sigma^* : w = plq, prq = w'$

Summary

- ▶ these are basic concepts, cf. also Baader/Nipkow
- ▶ you should work on the online exercises that require you to find derivations
- ▶ there is a highscore ranking: longer derivations are better (this is preparation for derivational complexity)