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On multiplier modules of Hilbert C*-modules

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Motivations from groupoid theory …

Roughly speaking:

If G and H are Morita equivalent as groupoids, then $C^*_r(G)$ and $C^*_r(H)$ are strongly Morita equivalent C*-algebras,

as well as C*(G) and C*(H) are strongly Morita equivalent C*-algebras.

Morita equivalence of C*-algebras preserves quite a number of properties of both the C*-algebras. Some of them can be reinterpreted as groupoid properties.

There is no transfer to Morita equivalence bimodules of the respective (local) multiplier algebras!

Extending these ideas beyond the (co)actions of locally compact groups on C*-algebras to the actions of locally compact groupoids G with Haar systems on C*-algebras A one can associate groupoid crossed products, which are also C*-algebras.

Strong Morita equivalences of C*-algebras A, B with (co)actions of such groupoids give rise to $A \times_{\delta_A} G$-$B \times_{\delta_B} G$ imprimitivity bimodules for the (full, reduced) groupoid crossed product C*-algebras. [Ren1-4], [leGall], [BaajSkan], [EchRae], [KS], [vEW], ..

So their ideal structure can be studied and compared in detail.

Another variant are actions by inverse semigroups S and by associated locally compact r-discrete groupoids G_S on C*-algebras.

They induce several C*-algebra isomorphisms and strong Morita equivalences of derived crossed product C*-algebras.

Sometimes actions by certain Hilbert $M_{(loc)}(A)$-modules are considered related to injective *-isomorphisms of one C*-algebra A into $M_{(loc)}(B)$ of another C*-algebra B.

So the correlated ideal structures can be studied.

Here again come (local) multiplier modules into play.

[Ren3], [KS], [KM1-4], [BKMS], [AKM], [M], [Tay1-2] …

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Resources in groupoid investigations (I):

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Settings …

Hilbert C*-modules (full – most offen today)

Let A be a C*-algebra and X be a complex-linear space and (right) A-module where both the complex-linear structures are compatible.

Let X admit a map $\langle .,. \rangle : X \times X \rightarrow A$ conjugate-A-linear in the first and A-linear in the second variable such that

(i) $\langle x,y \rangle = \langle y,x \rangle^*$ for any x,y in M,

(ii) $0 \leq \langle x,x \rangle$ for any x in M,

(iii) $\langle x,x \rangle = 0$ iff $x=0$.

Then $\|x\| := \|\langle x,x \rangle\|_A^{1/2}$ defines an A-module norm on X.

Examples .

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Operators on Hilbert C*-modules

cf. [28]

$K_A(X)$ - the C*-algebra of all "compact" A-linear operators on X, norm-closed linear hull of elementary operators $\{\theta_{x,y} : \theta_{x,y}(z) = y \cdot \langle x,z \rangle$ with z in $X\}$

$End_A^*(X)$ - the C*-algebra of all bounded adjointable A-linear operators on X $= M(K_A(X))$

$End_A(X)$ - the Banach algebra of all bounded A-linear operators on X $= LM(K_A(X))$

$End_A(X,X')$ - the Banach space of all bounded A-linear operators from X to X′ $= QM(K_A(X))$

The double centralizer approach to multiplier modules starts with two strongly Morita equivalent C*-algebras A, B and an A-B imprimitivity module X connecting them. ($B=K_A(X)$, $A=K_B(X)$.)

A *multiplier of X* is a pair $m=(m_A,m_B)$ of module maps, m_A: A → X, m_B: X → B such that

$m_A(a)\cdot b= a\cdot m_B(b)$ for any a in A, b in B.

Denotation of the multiplier module of X : M(X) .

X is isometrically A-B-bilinearly embeddable into M(X).

Facts (see [EchRae]):

M(X) is an A-B bimodule satisfying two conditions:

(i) $A \cdot M(X) \subseteq X$, $M(X) \cdot B \subseteq X$.

(ii) Any other A-B bimodule containing X and satisfying (i) admits an A-B bimodule homomorphism into M(X) acting as an identity map on X.

Two A-B bimodules satisfying (i)+(ii) are isomorphic to M(X) (uniqueness).

$End_A^*(A,X) = M(X) = End_B^*(X,B)$ $(m_A \leftarrow m \rightarrow m_B)$

(remember adjointability)

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The Linking algebra picture:

$$L = \begin{pmatrix} A & X \\ \tilde{X} & B \end{pmatrix}$$

with operations coming from module actions and C*-valued inner products on X .

$$M(L(X)) = \begin{pmatrix} M(A) & M(X) \\ M(\tilde{X}) & M(B) \end{pmatrix}, \text{ but } L(M(X)) \subseteq M(L(X)) \,!$$

Nevertheless, the upper right corner of M(L(X)) is unitarily isomorphic to M(X) . And M(L(X)) might not be a linking algebra.

So M(X) can be a non-projective left or right module!

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Example:

$A = \mathbb{C}$, $X = l_2$ – sep. Hilbert space, $B = K_{\mathbb{C}}(l_2)$

$M(X) = X$ – in this case, $L(M(X)) = \begin{pmatrix} \mathbb{C} & l_2 \\ l_2 & K_{\mathbb{C}}(l_2) \end{pmatrix} = L(X)$

$M(L(X)) = \begin{pmatrix} \mathbb{C} & l_2 \\ l_2 & B_{\mathbb{C}}(l_2) \end{pmatrix}$

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Example

$$A = \begin{pmatrix} c & c_0 \\ c_0 & c_0 \end{pmatrix}$$

$$M(A) = \begin{pmatrix} c & c_0 \\ c_0 & l_\infty \end{pmatrix}, \quad LM(A) = RM(A)^* = \begin{pmatrix} c & l_\infty \\ c_0 & l_\infty \end{pmatrix},$$

$$QM(A) = \begin{pmatrix} c & l_\infty \\ l_\infty & l_\infty \end{pmatrix}.$$

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Example (continued)

Consider A as a Hilbert A-module over itself in two ways:

$$T := \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in LM(A) \setminus M(A)$$

$\langle a,b\rangle_A$:= a*b for a,b in A

$\langle a,b\rangle_1$:= a*T*Tb = $\langle T(a),T(b)\rangle_A$ for a,b in A

One can calculate:

$$\left\langle \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \circ \begin{pmatrix} c & c_0 \\ c_0 & l_\infty \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \circ \begin{pmatrix} c & c_0 \\ c_0 & l_\infty \end{pmatrix} \right\rangle_A = \begin{pmatrix} c & c \\ c & l_\infty \end{pmatrix} \not\subseteq M(A) .$$

Example (continued)

Then:

$A = K_A(A) \neq K_{A,1}(A)$ *-isomorphically as C*-algebras

$M(A) = End_A^*(A) \neq End_{A,1}^*(A)$ *-isomorphically as C*-algebras

cf. [11].

Either M(A)=LM(A) and LM(A)=QM(A),

or $M(A) \subset LM(A) \subset QM(A)$, cf. [8, Cor. 4.18].

Multiplier modules of Hilbert C*-modules

There is a **second / third *approach*** to a theory of multiplier modules of Hilbert C*-modules.

I would like to stay with the approach by Damir Bakić and Boris Guljaš (2003) in [5,6] which focusses on a given C*-algebra A and on a given *full Hilbert A-module* {X, <.,.>} where the A-valued inner product <.,.> on X is considered within its class of *unitary equivalence*.

This allows to rely on the notions of "compact" and adjointable operators without reference to possibly existing not unitary equivalent inner products on X inducing equivalent norms. $K_A(X)$ becomes unique.

Multiplier modules ([5,6])

Let {X, <.,.>} be a full Hilbert C*-module over a given (non-unital) C*-algebra A . An extension of X is a triple (Y,B,Φ) such that

(i) B is a C*-algebra containing A as a two-sided norm-closed ideal.

(ii) Y is a Hilbert B-module.

(iii) $\Phi : X \to Y$ is a bounded module map satisfying $\langle \Phi(x), \Phi(y) \rangle_Y = \langle x,y \rangle_X$ for any x,y in X.

(iv) $Im(\Phi) = Y \cdot A$ = {za : z in Y, a in A} = {x in Y : <x,x> in A}

The triple (Y,B,Φ) is an essential extension of X, if A is an essential ideal of B. Then M(X) is the maximal one.

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Multiplier modules ([5,6])

Let $\{X, \langle .,. \rangle\}$ be a (not necessarily full) Hilbert C*-module over a given (non-unital) C*-algebra A. Denote by M(X) the set of all adjointable maps from A to X, i.e. $M(X) := End_A^*(A,X)$.

Obviously, M(X) is a Hilbert M(A)-module with the M(A)-valued inner product $\langle z_1, z_2 \rangle = z_1^* z_2$ for z_1, z_2 in M(X). The resulting Hilbert M(A)-module norm coincides with the operator norm on M(X).

We call M(X) the **multiplier module** of X. It is a maximal extension of X, in case X is full. $X = K_A(A,X)$.

Pairings and their uniqueness (I)

Proposition:

For a given pair of C*-algebras $(A,M(A))$, let X_1, X_2 be two full Hilbert C*-modules over A such that their multiplier modules $M(X_1)$, $M(X_2)$ are isometrically isomorphic as Hilbert $M(A)$-modules. Then X_1 and X_2 are isometrically isomorphic as Hilbert A-modules to $M(X_1)A = M(X_2)A$ resp..

So, the pairings $(X,M(X))$ are bound to each other for given C*-algebras $(A,M(A))$ up to unitary equivalence.

Pairings and their uniqueness (II)

Proposition:

Suppose, we have two non-isomorphic C*-algebras A_1 and A_2 such that they admit the same multiplier C*-algebra $M(A)$. Let X_1 be a full Hilbert A_1-module and X_2 a full Hilbert A_2-module such that $M(X_1)$ and $M(X_2)$ are unitarily isomorphic as Hilbert M(A)-modules.

Then X_1 is not unitarily isomorphic to X_2 as a Hilbert M(A)-module ($M(X_1)A_1=X_1$, $M(X_2)A_2=X_2$).

Topological characterization

Let A be a C*-algebra and X be a Hilbert A-module. Let a ("the") strict topology on M(X) be induced jointly by the two families of semi-norms {||za|| : a in A} and {||<z,x>|| : x in X, ||x||≤1} for z in M(X) . It is a locally convex topology.

The multiplier module M(X) turns out to be complete with respect to this strict topology, and M(X) is the strict completion of X , [5, Thm. 1.8, 1.9].

Moreover, the strict completion is an idempotent operation, i.e. $M_{M(A)}(M_A(X)) = M_A(X)$.

A Morita-like point of view

Theorem: cf. [EchRae, Prop. 1.3]

Let A be a C*-algebra and M(A) be its multiplier algebra. Let X be a full (right) Hilbert A-module and M(X) be its multiplier module, a full (right) Hilbert M(A)-module.

Then M(X) is also the full (left) multiplier module of the (left) Hilbert $K_A(X)$-module X with respect to the pairing of C*-algebras $K_A(X)$ and $M(K_A(X)) = End_A^*(X) = M(K_{M(A)}(M(X))) = End_{M(A)}^*(M(X))$, and vice versa.

There are non-unital, strongly Morita equivalent C*-algebras without multiplier imprimitivity bimodules.

-Isomorphism of key operator C-algebras

[5, Thm. 2.3].

For X,Y Hilbert A-modules each operator T in $End_A^*(X,Y)$ has an extension T_M in $End_{M(A)}^*(M(X),M(Y))$ with the same norm value obtained as the strict continuation of T . Therefore, it is uniquely determined.

Moreover, every operator in $End_{M(A)}^*(M(X),M(Y))$ arises this way, i.e. the C*-algebras $End_{M(A)}^*(M(X),M(Y))$ and $End_A^*(X,Y)$ are *-isomorphic.

This covers also the non-full variant of the definition.

"Compact" operator C*-algebras

Theorem:

Let A be a C*-algebra with multiplier algebra $M(A)$. Let X be a full Hilbert A-module and $M(X)$ be its full multiplier module.

The C*-algebra $K_A(X)$ of all "compact" operators on X admits a *-isomorphic embedding into the C*-algebra $K_{M(A)}(M(X))$ of all "compact" operators on $M(X)$.
$K_A(X)$ is smaller than $K_{M(A)}(M(X))$ if $X \neq M(X)$.

Bounded module operator algebras

Theorem:

Let A be a C*-algebra with multiplier algebra M(A) . Let X be a full Hilbert A-module and M(X) be its full multiplier module.

There does not exist any bounded M(A)-linear map $T_0 : M(X) \to M(X)$ such that $T_0 \neq 0$ on M(X), but $T_0 = 0$ on $X \subset M(X)$.

Bounded module operator algebras

Theorem (continued):

The Banach algebra $End_{M(A)}(M(X))$ admits an isometric embedding into the Banach algebra $End_A(X)$ by restricting an element on the domain from $M(X)$ to $X \subset M(X)$.

If the left multiplier algebra of $K_A(X)$ is larger than the multiplier algebra of it, then $End_{M(A)}(M(X))$ can be smaller than $End_A(X)$, i.e. not every bounded module operator might admit a bounded module operator continuation. (Existing continuations are unique.)

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Bounded module functionals

Theorem:

Let A be a C*-algebra with multiplier algebra M(A). Let X be a full Hilbert A-module and M(X) be its full multiplier module.

There does not exist any bounded M(A)-linear map $f_0 : M(X) \to M(A)$ such that $f_0 \neq 0$ on M(X), but $f_0 = 0$ on $X \subset M(X)$.

The Banach M(A)-module $M(X)'_{M(A)}$ admits an isometric modular embedding into the Banach A-module X'_A by restricting an element on the domain from M(X) to $X \subset M(X)$. There exist examples such that X'_A is strictly larger than the embedded copy of $M(X)'_{M(A)}$.

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Discussions …

Thank you for your attention.