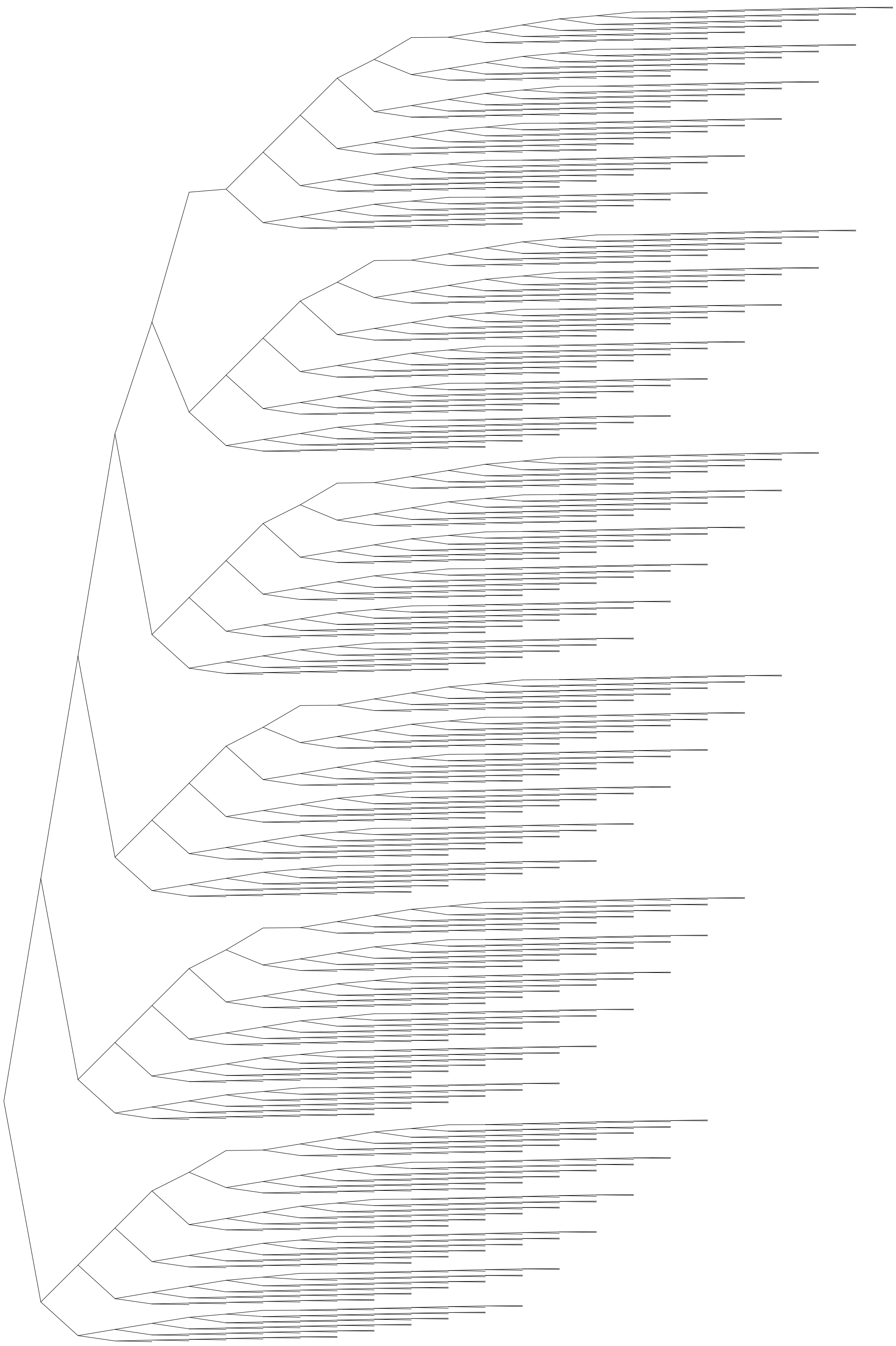
Part III

Aftermath



Normal form of the monster **S** A (**S S S S S S**) **S S S**
reached after 55 innermost reductions

Chapter 10

Conclusion

This thesis presented a detailed analysis of some properties of the system **CL**(**S**): absence of ground loops, top-termination, decidability of normalization, rationality of normalizing terms.

Absence of ground loops is shown by employing standard techniques: labeling and recursive path orders. So the proof looks quite nice and abstract.

The other results are proved by rather technical manipulations. In the case of top termination and decidability of normalization, these can still all be done and checked by hand, although this is a time-consuming process.

To verify the grammar that gives the rational language of normalizing expressions, the software `RX` was employed. The theory underlying that system has been described, and its usage is detailed in an appendix to this thesis.

Of course decidability of normalization follows from rationality of normalizing terms. Yet we chose to present these results separately because one depends on machine verification, and the other does not.

All of the proofs use the fact that **CL**(**S**) is an orthogonal, non-erasing term rewriting system. On the other hand they depend on the structure of the **S** rule. However, these dependencies are implicit, and scattered through all of the text. Therefore, for instance, the regular expressions that describe normalization look rather intricate, and they seem to appear *out of the blue*. It remains to find a general argument that tells *why* rational languages can describe **CL**(**S**), and a construction that *synthesizes* these languages, just given the **S** rule as input.

On one hand, the situation is aesthetically intriguing: a very rich structure arises from quite simple boundary conditions — just one, small rewrite rule. On the other hand, "surprising" answers indicate that the question wasn't asked in the right language. This thesis sheds some new light on **CL**(**S**) but this system remains something far from completely understood. The most intriguing open question is that of decidability of the word problem. We indicated how this might be attacked, using descriptions of infinite normal forms by finite automata.

We hope that further research into **CL**(**S**) will solve the word problem, and will also better explain the structure of the results that were obtained up to now, and lead to generalizations that are applicable to a larger class of rewrite systems.

Chapter 11

Bibliography

[ADN92] A. Arnold, A. Dicky, and M. Nivat. A note about minimal non-deterministic automata. *Bulletin of the EATCS*, 47:166–169, 1992.

[AGM92] S. Abramsky, D. Gabbay, and T. Maibaum, editors. *Handbook of Logic in Computer Science*, volume 2. Clarendon Press, Oxford, 1992.

[Aug96] Lennart Augustson. *The HBC compiler, Documentation for Release 0.9999.3*. Department of Computer Science, Chalmers University of Technology, Göteborg, ftp://ftp.cs.chalmers.se/pub/haskell/chalmers/, 1996.

[Ave95] Jürgen Avenhaus. *Reduktionssysteme*. Springer, 1995.

[Bar84] H. P. Barendregt. *The Lambda Calculus, Its Syntax and Sematics*. Elsevier Science Publishers, 1984.

[BK79] Jan Bergstra and Jan Willem Klop. Church–Rosser strategies in the Lambda calculus. *Theoretical Computer Science*, 9:27–38, 1979.

[CDGV91] J. L. Coquideé, M. Dauchet, R. Gilleron, and S. Vágvölgyi. Bottom-up tree pushdown automata and rewrite systems. In Dershowitz [Der91], pages 287–298.

[Chu41] Alonzo Church. *The Calculi of Lambda Conversion*, volume 6 of *Annals of Mathematics Studies*. Princeton University Press, 1941.

[Der91] N. Dershowitz, editor. *Rewriting Techniques and Applications*, number 488 in Lecture Notes in Computer Science. Springer Verlag, 1991.

[DJ90] Nachum Dershowitz and Jean-Pierre Jouannaud. *Rewrite Systems*, pages 243–319. Volume B of van Leeuwen [vL90], 1990.

[DJ91] Nachum Dershowitz and Jean-Pierre Jouannaud. Notations for rewriting. *Bull. EATCS*, 43:162–172, February 1991.

[DKP91] Nachum Dershowitz, Stéfane Kaplan, and David A. Plaisted. Rewrite, rewrite, rewrite, rewrite, rewrite, ... *Thoeretical Computer Science*, 83:71–96, 1991.

[DT90] M. Dauchet and S. Tison. The first order theory of ground rewriting systems is decidable. In LICS [LIC90], pages 242–248.

[ELS95] Uffe H. Engberg, Kim G. Larsen, and Arne Skou, editors. *Workshop on Tools and Algorithms for the Construction and Analysis of Systems (Aarhus, Denmark)*, number NS-95-2 in Notes Series, Department of Computer Science University of Aarhus, May 1995. BRICS.

[Gen97] Thomas Genet. Proving termination of sequential reduction relation using tree automata. talk given at 3rd International Workshop on Termination, Ede, The Netherlands, May 1997.

[Gra92] Bernhard Gramlich. Relating innermost, weak, uniform and modular termination of term rewriting systems. In Voronkov [Vor92], pages 285–296.

[GS84] F. Gécseg and M. Steinby. *Tree Automata.* Akadémiai Kiadó, Budapest, 1984.

[GT95] Rémi Gilleron and Sophie Tison. Regular tree langugages and rewrite systems. *Fund. Inf.*, 24:157–175, 1995.

[HH92] Dieter Hofbauer and Maria Huber. Computing linearizations using test sets. In Rusinowitch and Rémy [RR92], pages 287–301.

[Hin77] Roger Hindley. Combinatory reductions and lambda reductions compared. *Zeitschr. f. math. Logik und Grundlagen d. Math.*, 23:169–180, 1977.

[HS86] J. Roger Hindley and Jonathan P. Seldin. *Introduction to Combinators and λ-calculus.* Cambridge University Press, 1986.

[Jon96] Mark P. Jones. *Hugs 1.3, The Haskell User's Gofer System.* Department of Computer Science, University of Nottingham, ftp://ftp.cs.nott.ac.uk/pub/haskell/hugs/, 1996.

[KKSdV95] Richard Kennaway, Jan Willem Klop, Ronan Sleep, and Fer-Jan de Vries. Transfinite reductions in orthogonal term rewriting systems. *Information and Computation*, 119(1):18–38, May 1995.

[Klo80] Jan Willem Klop. *Reduction Cycles in Combinatory Logic*, pages 193–214. In Seldin and Hindley [SH80], 1980.

[Klo92] Jan Willem Klop. *Term Rewriting Systems*, pages 2–117. Volume 2 of Abramsky et al. [AGM92], 1992.

[Knu92] Donald Erwin Knuth. *Literate Programming*, volume 27 of *CSLI Lecture Notes.* Center for the Study of Language and Information, Standford, 1992.

[KT92] Gregory Kucherov and Mohamed Tajine. Decidability of regularity and related properties of ground normal form languages. In Rusinowitch and Rémy [RR92], pages 272–286.

[Kur90] Winfried Kurth. *Termination und Konfluenz von Semi-Thue-Systemen mit nur einer Regel.* PhD thesis, Technische Universität Clausthal, 1990.

[KvOvR93] J. W. Klop, V. van Oostrom, and F. van Raamsdonk. Combinatory reduction systems: introduction and survey. CWI report CS-R9362, Centrum foor Wiskunde en Informatica, 1993.

[Lam94] Leslie Lamport. *LaTeX, A Document Preparation System.* Addison Wesley, 1994. second edition.

[LIC90] IEEE Computer Society Press. *Logic in Computer Science.* IEEE Computer Society Press, 1990.

[MP95] Oliver Matz and Andreas Potthoff. Computing small nondeterministic finite automata. In Engberg et al. [ELS95], pages 74–88.

[Nip95] Tobias Nipkow. Equational reasoning: Lecture notes for a course on term rewriting etc. http://hpbroy3.informatik.tu-muenchen.de/˜nipkow/misc/er.dvi.gz, 1995.

[Oya87] M. Oyamaguchi. The church-rosser property for ground term rewriting systems is decidable. *Theoretical Computer Science*, 49:43–79, 1987.

[Röj96] Niklas Röjemo. *nhc: a space-efficient Haskell compiler.* Department of Computer Science, Chalmers University of Technology, Göteborg, ftp://ftp.cs.chalmers.se/pub/haskell/nhc13/, 1996.

[RR92] M. Rusinowitch and J. L. Rémy, editors. *Conditional Term Rewriting Systems, Third International Workshop*, number 656 in Lecture Notes in Computer Science. Springer Verlag, 1992.

[Sal88] K. Salomaa. Deterministic tree pushdown automata and monadic tree rewriting systems. *J. Comp. System Sci.*, 37:367–394, 1988.

[SH80] J. P. Seldin and J. R. Hindley, editors. *To H. B. Curry: Essays on Combinatory Logic, Lambda Calculus and Formalism.* Academic Press, 1980.

[Smu85] Raymond Smullyan. *To Mock a Mockingbird: and other logic puzzles including an amazing adventure in combinatory logic.* Knopf, New York, 1985.

[Sta89] Richard Statmann. The word problem for Smullyan's lark combinator is decidable. *J. Symbolic Comput.*, 7:103–112, 1989.

[SWB93] M. Sprenger and M. Wymann-Böni. How to decide the lark. *Theoretical Computer Science*, 110:419–432, 1993.

[Tea96] The GHC Team. *The Glorious Glasgow Haskell Compilation System, Version 2.01, User's Guide.* Department Of Computer Science, University of Glasgow, ftp://ftp.dcs.gla.ac.uk/pub/haskell/glasgow/, 1996.

[vdPSvdM80] W. L. van der Poel, C. E. Schaap, and G. van der Mey. New arithemtical operators in the theory of combinators. *Indagationes Mathematicae*, 42 (3):271–325, September 1980.

[VG92] S. Vágvölgyi and R. Gilleron. For a rewrite system it is decidable whether the set of irreducible ground terms is recognizable. *Bulletin of the EATCS*, 48:197–209, 1992.

[vL90] Jan van Leeuwen, editor. *Handbook of Theoretical Computer Science*, volume B. Elsevier Science Publishers B.V., 1990.

[Vor92] A. Voronkov, editor. *International Conference on Logic Programming and Automated Reasoning, St. Petersburg*, number 624 in Lecture Notes in Artificial Intelligence. Springer Verlag, 1992.

[Wal95] J. Waldmann. Typesetting haskell with LaTeX. Forschungsergebnisse Math/95/9, Friedrich-Schiller-Universität Jena, Fakultät für Mathematik und Informatik, 1995.

[Wal96] J. Waldmann. Infinitite unique head reductions for $Sxyz \to xz(yz)$. Forschungsergebnisse 96/27, Friedrich-Schiller-Universität Jena, Fakultät für Mathematik und Informatik, 1996.

[Wal97] Johannes Waldmann. *The Combinator S*. PhD thesis, Friedrich-Schiller-Universität Jena, 1997.

[Zac78] Efstathios Zachos. Kombinatorische Logik und S-Terme. Berichte des Instituts für Informatik 26, Eidgenössische Technische Hochschule Zürich, 1978.

[ZG96] H. Zantema and A. Geser. Non-looping rewriting. Technical Report UU-CS-1996-03, Utrecht University, January 1996.

Chapter 12

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